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On the classical string solutions and string/field theory duality

Desislava Aleksandrova and P. Bozhilov

Department of Theoretical and Applied Physics, Shoumen University
9712 Shoumen, Bulgaria
E-mail: p.bozhilov@shu-bg.net

Abstract: We classify almost all classical string configurations, considered in the framework of the semi-classical limit of the string/gauge theory duality. Then, we describe a procedure for obtaining the conserved quantities and the exact classical string solutions in general string theory backgrounds, when the string embedding coordinates depend non-linearly on the worldsheet time parameter.

Keywords: Bosonic Strings, AdS-CFT and dS-CFT Correspondence, Space-Time Symmetries, Integrable Equations in Physics.

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Contents

1. Introduction

After the appearance of the article [1] on the semi-classical limit of the gauge/string correspondence, a lot of papers have been devoted to the investigation of the connection between the classical string solutions, their semi-classical quantization and the string/field theory duality [2], [4]–[6], [8]–[15], [17]–[26]. Different string configurations have been considered: rotating, pulsating and orbiting strings. Most of the authors investigated the closed string case. In [20, 24] the open string case was considered, where nontrivial boundary conditions had to be also satisfied.

The most explored background was $AdS_5 \times S^5$. However, the string dynamics has been investigated in many other string theory backgrounds, known to have field theory dual descriptions in different dimensions, with different number of (or without) supersymmetries, conformal or non-conformal. The influence of the B-field on the obtained string solutions has been also considered. Besides, solutions for higher dimensional extended objects (M2-, D3- and M5-branes) have been obtained [3, 7, 11, 12, 16].

For establishing the correspondence between the semi-classically quantized string solutions and the appropriate objects in the dual field theory, it is essential for one to know the explicit expressions for the conserved charges like energy, spin, etc., on the string theory side. Their existence is connected with the symmetries of the corresponding supergravity backgrounds, generated by the Killing vectors for these backgrounds. The analysis of the connection between the ansatzes, used to obtain exact string solutions, and the background symmetries shows that the latter have been essentially explored in the process of solving the string equations of motion and constraints.

The comparison of the ansatzes used in [1, 2], [4]–[6], [8]–[15], [17]–[26], with the corresponding symmetries of the target space-times, led us to the following classification scheme:

$$\begin{aligned}
X^\mu(\tau,\sigma) &= \Lambda_0^\mu\tau + \Lambda_1^\mu\sigma\,, & X^a(\tau,\sigma) &= Y^a(\tau)\,; & (1.1)\\
X^\mu(\tau,\sigma) &= \Lambda_0^\mu\tau + \Lambda_1^\mu\sigma + Y^\mu(\tau)\,, & X^a(\tau,\sigma) &= Y^a(\tau)\,; & (1.2)\\
X^\mu(\tau,\sigma) &= \Lambda_0^\mu\tau + \Lambda_1^\mu\sigma\,, & X^a(\tau,\sigma) &= Z^a(\sigma)\,; & (1.3)\\
X^\mu(\tau,\sigma) &= \Lambda_0^\mu\tau + \Lambda_1^\mu\sigma + Z^\mu(\sigma)\,, & X^a(\tau,\sigma) &= Z^a(\sigma)\,; & (1.4)\\
\Lambda_m^\mu &= \text{const}\,, \qquad (m=0,1)\,.
\end{aligned}$$

In the above equalities, the string embedding coordinates $X^M(\tau,\sigma)$, $(M = 0, 1, \ldots, D-1)$, are divided into coordinates $X^\mu(\tau,\sigma)$ and $X^a(\tau,\sigma)$, such that

$$\dim\{\mu\} + \dim\{a\} = \dim\{M\}\,.$$

$X^\mu(\tau,\sigma)$ correspond to the space-time coordinates x^μ, on which the background fields do not depend. In other words, there exist $dim\{\mu\}$ commuting Killing vectors $\partial/\partial x^\mu$. In (1.1)–(1.4), we have separated the cases $Y^\mu = 0$ and $Y^\mu \neq 0$, $Z^\mu = 0$ and $Z^\mu \neq 0$, because the types of the solutions in these cases are essentially different, as we will see later on.

All the ansatzes used in [1, 2], [4]–[6], [8]–[15], [17]–[26], are particular cases of (1.1)–(1.4), except two [19]. In [8, 11, 17, 25] there are of type (1.1), in [1] and [9] — of type (1.2). In [1, 2], [4]–[6], [9, 10, 12, 13], [17]–[23], [26], ansatzes of the type (1.3) are used. Solutions, based on the ansatzes of the type (1.4), are obtained in [14, 15, 17, 24].

The aim of this article is to describe a procedure for obtaining the conserved quantities and the exact classical string solutions in general string theory backgrounds, based on the ansatzes (1.1) and (1.2). Besides, we will use more general worldsheet gauge than the conformal one, in order to be able to discuss the tensionless limit $T \to 0$, corresponding to small t'Hooft coupling $\lambda \to 0$ on the field theory side. The zero tension limit is also interesting in connection with the ongoing discussion on massless higher spin field theories.

The paper is organized as follows. In section 2 and section 3, we describe the string dynamics and obtain the corresponding exact solutions, based on the ansatzes (1.1) and (1.2) respectively. Section 4 is devoted to some applications of the obtained general results.

2. Exact string solutions

The Polyakov type action for the bosonic string in a D-dimensional curved space-time with metric tensor $g_{MN}(x)$, interacting with a background 2-form gauge field $b_{MN}(x)$ via Wess-Zumino term, can be written as

$$\begin{aligned}
S^P &= -\frac{1}{2}\int d^2\xi \left\{ T\sqrt{-\gamma}\left[\gamma^{mn}\partial_m X^M \partial_n X^N g_{MN}\right] - Q\varepsilon^{mn}\partial_m X^M \partial_n X^N b_{MN}\right\},\\
\partial_m &= \frac{\partial}{\partial\xi^m}\,, \qquad \xi^m = (\xi^0,\xi^1) = (\tau,\sigma)\,, \qquad m,n = 0,1; \qquad M,N = 0,1,\ldots,D-1\,, \quad (2.1)
\end{aligned}$$

where γ is the determinant of the auxiliary worldsheet metric γ_{mn}, and γ^{mn} is its inverse. The position of the string in the background space-time is given by $x^M = X^M(\xi^m)$, and $T =$

$1/2\pi\alpha'$, Q are the string tension and charge, respectively. If we consider the action (2.1) as a bosonic part of a supersymmetric one, we have to set $Q = \pm T$. In what follows, $Q = T$.

The action (2.1) is known to be classically equivalent to the Nambu-Goto type action:

$$S^{NG} = -T \int d^2\xi \left[\sqrt{-G} - \frac{1}{2}\varepsilon^{mn}\partial_m X^M \partial_n X^N b_{MN}(X)\right],$$

where $G \equiv \det(G_{mn})$ and

$$G_{mn} = \partial_m X^M \partial_n X^N g_{MN}(X)$$

is the metric induced on the string worldsheet. We will work with the Polyakov type action.

The equations of motion for X^M following from (2.1) are:

$$-\, g_{LK}\left[\partial_m\left(\sqrt{-\gamma}\gamma^{mn}\partial_n X^K\right) + \sqrt{-\gamma}\gamma^{mn}\Gamma^K_{MN}\partial_m X^M \partial_n X^N\right] = \frac{1}{2}H_{LMN}\epsilon^{mn}\partial_m X^M \partial_n X^N\,, \tag{2.2}$$

where

$$\begin{aligned}\Gamma_{L,MN} &= g_{LK}\Gamma^K_{MN} = \frac{1}{2}\left(\partial_M g_{NL} + \partial_N g_{ML} - \partial_L g_{MN}\right),\\ H_{LMN} &= \partial_L b_{MN} + \partial_M b_{NL} + \partial_N b_{LM}\,.\end{aligned}$$

The constraints are obtained by varying the action (2.1) with respect to γ^{mn}:

$$\delta_{\gamma^{mn}} S^P = 0 \Rightarrow \left(\partial_m X^M \partial_n X^N - \frac{1}{2}\gamma_{mn}\gamma^{kl}\partial_k X^M \partial_l X^N\right) g_{MN} = 0\,. \tag{2.3}$$

Now, we would like to solve (2.2) and (2.3). Let us first consider the constraints (2.3). In order to work with γ^{mn} only, we rewrite them as

$$\left(\gamma^{kl}\gamma^{mn} - 2\gamma^{km}\gamma^{ln}\right) G_{mn} = 0\,. \tag{2.4}$$

We have three constraints in (2.4), but only two of them are independent. To extract the independent ones, we rewrite the three constraints as follows:

$$\left(\gamma^{00}\gamma^{mn} - 2\gamma^{0m}\gamma^{0n}\right) G_{mn} = 0\,, \tag{2.5}$$
$$\left(\gamma^{01}\gamma^{mn} - 2\gamma^{0m}\gamma^{1n}\right) G_{mn} = 0\,, \tag{2.6}$$
$$\left(\gamma^{11}\gamma^{mn} - 2\gamma^{1m}\gamma^{1n}\right) G_{mn} = 0\,. \tag{2.7}$$

Inserting G_{00} from (2.5) into (2.6) and (2.7), one obtains that both of them are satisfied, when the equality

$$\gamma^{00}G_{01} + \gamma^{01}G_{11} = 0 \tag{2.8}$$

is fulfilled. To simplify the constraint (2.5), we put (2.8) in it, which results in

$$\gamma^{00}G_{00} - \gamma^{11}G_{11} = 0\,. \tag{2.9}$$

So, our *independent* constraints, with which we will work from now on, are given by (2.8) and (2.9).

Now let us turn to the equations of motion (2.2). We will work in the gauge $\gamma^{mn} =$ *constants*, in which they simplify to

$$-\sqrt{-\gamma}\gamma^{mn}g_{LK}\left[\partial_m\partial_n X^K + \Gamma^K_{MN}\partial_m X^M\partial_n X^N\right] = \frac{1}{2}H_{LMN}\epsilon^{mn}\partial_m X^M\partial_n X^N\,. \tag{2.10}$$

In particular, $\gamma^{mn} = \eta^{mn} = diag(-1,1)$ correspond to the the usually used *conformal gauge*.

2.1 Solving the equations of motion and constraints

In the frequently used *static gauge*, one makes the following identification: $X^m(\xi^n) = \xi^m$. It is applied to fix the gauge freedom due to the invariance of the action (2.1) with respect to infinitesimal diffeomorphisms (reparametrizations) of the string worldsheet. Instead, we will use a more general gauge than the static one. It exploits the symmetry of the background, which exists for every physically relevant external fields. Namely, our ansatz for the string coordinates $X^M = (X^\mu, X^a)$ is given by (1.1), and x^μ are the target space-time coordinates, on which the background fields do not depend:

$$\partial_\mu g_{MN} = 0, \qquad \partial_\mu b_{MN} = 0\,. \tag{2.11}$$

If we restrict ourselves to $\mu = m$ and $\Lambda^m_n = \delta^m_n$, we come back to static gauge.[1].

Taking into account the ansatz (1.1), the lagrangian density, the induced metric, the constraints (2.9) and (2.8) respectively, and the Euler-Lagrange equations for X^M (2.10), can be written as (the over-dot is used for $d/d\tau$)

$$\begin{aligned}
\mathcal{L}^A(\tau) = -\frac{T}{2}\sqrt{-\gamma}\Bigg[&\gamma^{00}g_{ab}\dot{Y}^a\dot{Y}^b + 2\left(\gamma^{0n}g_{a\nu}\Lambda^\nu_n - \frac{1}{\sqrt{-\gamma}}\Lambda^\nu_1 b_{a\nu}\right)\dot{Y}^a + \\
&+\gamma^{mn}\Lambda^\mu_m\Lambda^\nu_n g_{\mu\nu} - \frac{2}{\sqrt{-\gamma}}\Lambda^\mu_0\Lambda^\nu_1 b_{\mu\nu}\Bigg]\,;
\end{aligned} \tag{2.12}$$

$$G_{00} = g_{ab}\dot{Y}^a\dot{Y}^b + 2\Lambda^\nu_0 g_{\nu a}\dot{Y}^a + \Lambda^\mu_0\Lambda^\nu_0 g_{\mu\nu}\,, \tag{2.13}$$

$$G_{01} = \Lambda^\nu_1\left(g_{\nu a}\dot{Y}^a + \Lambda^\mu_0 g_{\mu\nu}\right), \qquad G_{11} = \Lambda^\mu_1\Lambda^\nu_1 g_{\mu\nu}\,;$$

$$\gamma^{00}g_{ab}\dot{Y}^a\dot{Y}^b + 2\gamma^{00}\Lambda^\nu_0 g_{\nu a}\dot{Y}^a + \left(\gamma^{00}\Lambda^\mu_0\Lambda^\nu_0 - \gamma^{11}\Lambda^\mu_1\Lambda^\nu_1\right)g_{\mu\nu} = 0\,, \tag{2.14}$$

$$\Lambda^\nu_1\left(\gamma^{00}g_{\nu a}\dot{Y}^a + \gamma^{0n}\Lambda^\mu_n g_{\mu\nu}\right) = 0\,; \tag{2.15}$$

$$\begin{aligned}
&\gamma^{00}\left(g_{Lb}\ddot{Y}^b + \Gamma_{L,bc}\dot{Y}^b\dot{Y}^c\right) + 2\gamma^{0n}\Lambda^\mu_n\Gamma_{L,\mu b}\dot{Y}^b + \gamma^{mn}\Lambda^\mu_m\Lambda^\nu_n\Gamma_{L,\mu\nu} = \\
&= -\frac{1}{\sqrt{-\gamma}}\Lambda^\nu_1\left(H_{L\mu\nu}\Lambda^\mu_0 + H_{La\nu}\dot{Y}^a\right).
\end{aligned} \tag{2.16}$$

$\mathcal{L}^A(\tau)$ in (2.12) is like a lagrangian for a point particle, interacting with the external fields g_{MN}, $b_{a\nu}$ and $b_{\mu\nu}$.

[1]We note however that this gauge has been never used when semi-classical string quantization was considered.

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Let us write down the conserved quantities. By definition, the generalized momenta are

$$P_L \equiv \frac{\partial \mathcal{L}}{\partial(\partial_0 X^L)} = -T\left(\sqrt{-\gamma}\gamma^{0n} g_{LN}\partial_n X^N - b_{LN}\partial_1 X^N\right).$$

For our ansatz, they take the form:

$$P_L = -T\left[\sqrt{-\gamma}\left(\gamma^{00} g_{La}\dot{Y}^a + \gamma^{0n} g_{L\nu}\Lambda_n^\nu\right) - b_{L\nu}\Lambda_1^\nu\right].$$

The lagrangian (2.12) does not depend on the coordinates X^μ. Therefore, the conjugated momenta P_μ are conserved

$$P_\mu = -T\left[\sqrt{-\gamma}\left(\gamma^{00} g_{\mu a}\dot{Y}^a + \gamma^{0n}\Lambda_n^\nu g_{\mu\nu}\right) - \Lambda_1^\nu b_{\mu\nu}\right] = \text{constants}.$$

The same result can be obtained by solving the equations of motion (2.16) for $L = \lambda$. In accordance with (2.11), the computation of $\Gamma_{\lambda,MN}$ and $H_{\lambda,MN}$ gives

$$\begin{aligned}
\Gamma_{\lambda,ab} &= \frac{1}{2}\left(\partial_a g_{b\lambda} + \partial_b g_{a\lambda}\right), & \Gamma_{\lambda,\mu a} &= \frac{1}{2}\partial_a g_{\mu\lambda}, & \Gamma_{\lambda,\mu\nu} &= 0, \\
H_{\lambda ab} &= \partial_a b_{b\lambda} + \partial_b b_{\lambda a}, & H_{\lambda\mu a} &= \partial_a b_{\lambda\mu}, & H_{\lambda\mu\nu} &= 0.
\end{aligned} \tag{2.17}$$

Inserting these expressions in the part of the differential equations (2.16) corresponding to $L = \lambda$, and using the equalities $\dot{g}_{MN} = \dot{Y}^a\partial_a g_{MN}$, $\dot{b}_{MN} = \dot{Y}^a\partial_a b_{MN}$, one receives the first integrals

$$\gamma^{00} g_{\lambda a}\dot{Y}^a + \gamma^{0n}\Lambda_n^\mu g_{\lambda\mu} - \frac{1}{\sqrt{-\gamma}}\Lambda_1^\nu b_{\lambda\nu} = \text{constants}.$$

It is easy to check that they are connected with the conserved momenta P_μ as

$$\gamma^{00} g_{\mu a}\dot{Y}^a + \gamma^{0n}\Lambda_n^\nu g_{\mu\nu} - \frac{1}{\sqrt{-\gamma}}\Lambda_1^\nu b_{\mu\nu} = -\frac{P_\mu}{T\sqrt{-\gamma}}. \tag{2.18}$$

From (2.15) and (2.18), one obtains the following compatibility condition

$$\Lambda_1^\nu P_\nu = 0. \tag{2.19}$$

This equality may be interpreted as a solution of the constraint (2.15), which restricts the number of the independent parameters in the theory.

With the help of (2.18), the other constraint, (2.14), can be rewritten in the form

$$g_{ab}\dot{Y}^a\dot{Y}^b = \mathcal{U}, \tag{2.20}$$

where $\mathcal{U}$ is given by

$$\mathcal{U} = \frac{1}{\gamma^{00}}\left[\gamma^{mn}\Lambda_m^\mu\Lambda_n^\nu g_{\mu\nu} + \frac{2\Lambda_0^\mu}{T\sqrt{-\gamma}}\left(P_\mu - T\Lambda_1^\nu b_{\mu\nu}\right)\right]. \tag{2.21}$$

Now, let us turn to the equations of motion (2.16), corresponding to $L = a$. By using the explicit expressions

$$\begin{aligned}
\Gamma_{a,\mu b} &= -\frac{1}{2}\left(\partial_a g_{b\mu} - \partial_b g_{a\mu}\right) = -\partial_{[a} g_{b]\mu}, & \Gamma_{a,\mu\nu} &= -\frac{1}{2}\partial_a g_{\mu\nu}, \\
H_{a\mu\nu} &= \partial_a b_{\mu\nu}; & H_{ab\nu} &= \partial_a b_{b\nu} - \partial_b b_{a\nu} = 2\partial_{[a} b_{b]\nu},
\end{aligned}$$

one obtains

$$g_{ab}\ddot{Y}^b + \Gamma_{a,bc}\dot{Y}^b\dot{Y}^c = \frac{1}{2}\partial_a\mathcal{U} + 2\partial_{[a}\mathcal{A}_{b]}\dot{Y}^b\,. \tag{2.22}$$

In (2.22), an effective potential $\mathcal{U}$ and an effective gauge field $\mathcal{A}_a$ appeared. $\mathcal{U}$ is given in (2.21), and

$$\mathcal{A}_a = \frac{1}{\gamma^{00}}\left(\gamma^{0m}\Lambda^\mu_m g_{a\mu} - \frac{\Lambda^\mu_1 b_{a\mu}}{\sqrt{-\gamma}}\right). \tag{2.23}$$

The reduced equations of motion (2.22) are as for a point particle moving in the gravitational field g_{ab}, in the potential $\mathcal{U}$ and interacting with the 1-form gauge field $\mathcal{A}_a$ through its field strength $\mathcal{F}_{ab} = 2\partial_{[a}\mathcal{A}_{b]}$.

Now our task is to find *exact* solutions of the *nonlinear* differential equations (2.20) and (2.22). It turns out that for background fields depending on only one coordinate x^a, we can always integrate these equations, and the solution is [2]

$$\tau\left(X^a\right) = \tau_0 \pm \int_{X_0^a}^{X^a} dx \left(\frac{\mathcal{U}}{g_{aa}}\right)^{-1/2}. \tag{2.24}$$

Otherwise, supposing the metric g_{ab} is a diagonal one, (2.22) and (2.20) reduce to

$$\frac{d}{d\tau}(g_{aa}\dot{Y}^a) - \frac{1}{2}\left[\partial_a g_{aa}(\dot{Y}^a)^2 + \partial_a\mathcal{U}\right] - \frac{1}{2}\sum_{b\neq a}\left[\partial_a g_{bb}(\dot{Y}^b)^2 + 4\partial_{[a}\mathcal{A}_{b]}\dot{Y}^b\right] = 0\,, \tag{2.25}$$

$$g_{aa}(\dot{Y}^a)^2 + \sum_{b\neq a} g_{bb}(\dot{Y}^b)^2 = \mathcal{U}\,. \tag{2.26}$$

With the help of the constraint (2.26), we can rewrite the equations of motion (2.25) in the form

$$\frac{d}{d\tau}(g_{aa}\dot{Y}^a)^2 - \dot{Y}^a\partial_a\left(g_{aa}\mathcal{U}\right) + \dot{Y}^a\sum_{b\neq a}\left[\partial_a\left(\frac{g_{aa}}{g_{bb}}\right)(g_{bb}\dot{Y}^b)^2 - 4g_{aa}\partial_{[a}\mathcal{A}_{b]}\dot{Y}^b\right] = 0\,. \tag{2.27}$$

To find solutions of the above equations without choosing particular background, we can fix all coordinates Y^a except one. Then the *exact* string solution of the equations of motion and constraints is given again by the same expression (2.24) for $\tau\left(X^a\right)$.

To find solutions depending on more than one coordinate, we have to impose further conditions on the background fields. Let us first consider the simpler case, when the last two terms in (2.27) are not present. This may happen, when

$$\partial_a\left(\frac{g_{aa}}{g_{bb}}\right) = 0\,, \qquad \mathcal{A}_a = 0\,. \tag{2.28}$$

Then, the first integrals of (2.27) are

$$\left(g_{aa}\dot{Y}^a\right)^2 = D_a(Y^{b\neq a}) + g_{aa}\mathcal{U}\,, \tag{2.29}$$

[2] In this case, the constraint (2.20) is first integral for the equation of motion (2.22).

where D_a are arbitrary functions of their arguments. These solutions must be compatible with the constraint (2.26), which leads to the condition

$$\sum_a \frac{D_a}{g_{aa}} = (1-n_a)\mathcal{U}\,,$$

where n_a is the number of the coordinates Y^a. From here, one can express one of the functions D_a through the others. To this end, we split the index a in such a way that Y^r is one of the coordinates Y^a, and Y^α are the others. Then

$$D_r = -g_{rr}\left(n_\alpha \mathcal{U} + \sum_\alpha \frac{D_\alpha}{g_{\alpha\alpha}}\right),$$

and by using this, one rewrites the first integrals (2.29) as

$$\left(g_{rr}\dot{Y}^r\right)^2 = g_{rr}\left[(1-n_\alpha)\mathcal{U} - \sum_\alpha \frac{D_\alpha}{g_{\alpha\alpha}}\right] \geq 0\,, \qquad \left(g_{\alpha\alpha}\dot{Y}^\alpha\right)^2 = D_\alpha(Y^{a\neq\alpha}) + g_{\alpha\alpha}\mathcal{U} \geq 0\,, \tag{2.30}$$

where n_α is the number of the coordinates Y^α. Thus, the constraint (2.26) is satisfied identically.

Now we turn to the general case, when all terms in the equations of motion (2.27) are present. The aim is to find conditions, which will allow us to reduce the order of the equations of motion by one. An example of such *sufficient* conditions, is given below :

$$\begin{aligned} \mathcal{A}_a \equiv (\mathcal{A}_r, \mathcal{A}_\alpha) &= (\mathcal{A}_r, \partial_\alpha f)\,, & \partial_\alpha\left(\frac{g_{\alpha\alpha}}{g_{aa}}\right) &= 0\,, \\ \partial_\alpha\left(g_{rr}\dot{Y}^r\right)^2 &= 0\,, & \partial_r\left(g_{\alpha\alpha}\dot{Y}^\alpha\right)^2 &= 0\,. \end{aligned} \tag{2.31}$$

By using the restrictions given above, one obtains the following first integrals of the equations (2.27), compatible with the constraint (2.26)

$$\left(g_{rr}\dot{Y}^r\right)^2 = g_{rr}\left[(1-n_\alpha)\,\mathcal{U} - \sum_\alpha \frac{D_\alpha}{g_{\alpha\alpha}} - 2n_\alpha\left(\mathcal{A}_r - \partial_r f\right)\dot{Y}^r\right] = E_r\left(Y^r\right) \geq 0\,, \tag{2.32}$$

$$\left(g_{\alpha\alpha}\dot{Y}^\alpha\right)^2 = D_\alpha\left(Y^{a\neq\alpha}\right) + g_{\alpha\alpha}\left[\mathcal{U} + 2\left(\mathcal{A}_r - \partial_r f\right)\dot{Y}^r\right] = E_\alpha\left(Y^\beta\right) \geq 0\,, \tag{2.33}$$

where D_α, E_α and E_r are arbitrary functions of their arguments.

Further progress is possible, when working with particular background configurations, allowing for separation of the variables in (2.30), or in (2.32) and (2.33).

2.2 The tensionless limit

Our results obtained so far are not applicable to tensionless (null) strings, because the action (2.1) is proportional to the string tension T. The parameterization of γ^{mn}, which is appropriate for considering the zero tension limit $T \to 0$, is the following [27, 28]:

$$\gamma^{00} = -1, \qquad \gamma^{01} = \lambda^1, \qquad \gamma^{11} = (2\lambda^0 T)^2 - (\lambda^1)^2, \qquad \det(\gamma^{mn}) = -(2\lambda^0 T)^2\,. \tag{2.34}$$

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Here λ^n are the Lagrange multipliers, whose equations of motion generate the *independent* constraints. In these notations, the constraints (2.14) and (2.15), the equations of motion (2.16), and the conserved momenta (2.18) take the form

$$\begin{aligned}
g_{ab}\dot{Y}^a\dot{Y}^b + 2\Lambda_0^\nu g_{\nu a}\dot{Y}^a + \left\{\Lambda_0^\mu\Lambda_0^\nu + \left[(2\lambda^0 T)^2 - (\lambda^1)^2\right]\Lambda_1^\mu\Lambda_1^\nu\right\} g_{\mu\nu} &= 0\,,\\
\Lambda_1^\nu\left[g_{\nu a}\dot{Y}^a + \left(\Lambda_0^\mu - \lambda^1\Lambda_1^\mu\right) g_{\mu\nu}\right] &= 0\,;\\
g_{Lb}\ddot{Y}^b + \Gamma_{L,bc}\dot{Y}^b\dot{Y}^c + 2\left(\Lambda_0^\mu - \lambda^1\Lambda_1^\mu\right)\Gamma_{L,\mu b}\dot{Y}^b + &\\
+\left[\left(\Lambda_0^\mu - \lambda^1\Lambda_1^\mu\right)\left(\Lambda_0^\nu - \lambda^1\Lambda_1^\nu\right) - (2\lambda^0 T)^2\Lambda_1^\mu\Lambda_1^\nu\right]\Gamma_{L,\mu\nu} &=\\
= 2\lambda^0 T\Lambda_1^\nu\left(H_{L\nu b}\dot{Y}^b + \Lambda_0^\mu H_{L\mu\nu}\right)&;\\
g_{\mu a}\dot{Y}^a + \left(\Lambda_0^\nu - \lambda^1\Lambda_1^\nu\right) g_{\mu\nu} + 2\lambda^0 T\Lambda_1^\nu b_{\mu\nu} &= 2\lambda^0 P_\mu\,.
\end{aligned}$$

The reduced equations of motion and constraint (2.22) and (2.20) have the same form, but now, the effective potential (2.21) and the effective gauge field (2.23) are given by

$$\begin{aligned}
\mathcal{U}^\lambda &= \left[\left(\Lambda_0^\mu - \lambda^1\Lambda_1^\mu\right)\left(\Lambda_0^\nu - \lambda^1\Lambda_1^\nu\right) - (2\lambda^0 T)^2\Lambda_1^\mu\Lambda_1^\nu\right] g_{\mu\nu} - 4\lambda^0\Lambda_0^\mu\left(P_\mu - T\Lambda_1^\nu b_{\mu\nu}\right),\\
\mathcal{A}_a^\lambda &= \left(\Lambda_0^\mu - \lambda^1\Lambda_1^\mu\right) g_{a\mu} + 2\lambda^0 T\Lambda_1^\mu b_{a\mu}\,.
\end{aligned}$$

If one sets $\lambda^1 = 0$ and $2\lambda^0 T = 1$, the results in *conformal gauge* are obtained, as it should be. If one puts $T = 0$ in the above formulas, they will describe *tensionless* strings.

3. Exact solutions for more general string embedding

Now, we are going to use the ansatz (1.2) for the string coordinates, which corresponds to more general string embedding. Here, compared with (1.1), X^μ are allowed to vary non-linearly with the proper time τ. In addition, we assume that the conditions (2.11) on the background fields still hold.

By using the ansatz (1.2), one obtains that the lagrangian density, the induced metric, the constraints (2.9) and (2.8) respectively, and the Euler-Lagrange equations for X^M (2.10) are given by

$$\begin{aligned}
\mathcal{L}^{GA}(\tau) = -\frac{T}{2}\sqrt{-\gamma}\Bigg[&\gamma^{00} g_{MN}\dot{Y}^M\dot{Y}^N + 2\left(\gamma^{0n}\Lambda_n^\nu g_{M\nu} - \frac{\Lambda_1^\nu b_{M\nu}}{\sqrt{-\gamma}}\right)\dot{Y}^M +\\
&+\gamma^{mn}\Lambda_m^\mu\Lambda_n^\nu g_{\mu\nu} - \frac{2\Lambda_0^\mu\Lambda_1^\nu b_{\mu\nu}}{\sqrt{-\gamma}}\Bigg]\,;
\end{aligned}$$

$$G_{00} = g_{MN}\dot{Y}^M\dot{Y}^N + 2\Lambda_0^\nu g_{\nu N}\dot{Y}^N + \Lambda_0^\mu\Lambda_0^\nu g_{\mu\nu}\,, \tag{3.1}$$

$$G_{01} = \Lambda_1^\nu\left(g_{\nu N}\dot{Y}^N + \Lambda_0^\mu g_{\mu\nu}\right), \qquad G_{11} = \Lambda_1^\mu\Lambda_1^\nu g_{\mu\nu}\,;$$

$$\gamma^{00} g_{MN}\dot{Y}^M\dot{Y}^N + 2\gamma^{00}\Lambda_0^\nu g_{\nu N}\dot{Y}^N + \left(\gamma^{00}\Lambda_0^\mu\Lambda_0^\nu - \gamma^{11}\Lambda_1^\mu\Lambda_1^\nu\right) g_{\mu\nu} = 0\,, \tag{3.2}$$

$$\Lambda_1^\nu\left(\gamma^{00} g_{\nu N}\dot{Y}^N + \gamma^{0n}\Lambda_n^\mu g_{\mu\nu}\right) = 0\,; \tag{3.3}$$

$$\begin{aligned}
&\gamma^{00}\left(g_{LN}\ddot{Y}^N + \Gamma_{L,MN}\dot{Y}^M\dot{Y}^N\right) + 2\gamma^{0n}\Lambda_n^\mu\Gamma_{L,\mu N}\dot{Y}^N + \gamma^{mn}\Lambda_m^\mu\Lambda_n^\nu\Gamma_{L,\mu\nu} = \\
&= -\frac{1}{\sqrt{-\gamma}}\Lambda_1^\nu\left(H_{LM\nu}\dot{Y}^M + \Lambda_0^\mu H_{L\mu\nu}\right).
\end{aligned} \tag{3.4}$$

The conserved momenta P_μ can be found as before, and now they are

$$\gamma^{00} g_{\mu N} \dot{Y}^N + \gamma^{0n} \Lambda_n^\nu g_{\mu\nu} - \frac{\Lambda_1^\nu b_{\mu\nu}}{\sqrt{-\gamma}} = -\frac{P_\mu}{T\sqrt{-\gamma}} = \text{constants} . \tag{3.5}$$

The compatibility condition following from the constraint (3.3) and from (3.5) coincides with the previous one (2.19). With the help of (3.5), the equations of motion (3.4) corresponding to $L = a$ and the other constraint (3.2), can be rewritten in the form

$$g_{aN} \ddot{Y}^N + \Gamma_{a,MN} \dot{Y}^M \dot{Y}^N = \frac{1}{2} \partial_a \mathcal{U} + 2 \partial_{[a} \mathcal{A}_{N]} \dot{Y}^N , \tag{3.6}$$

$$g_{MN} \dot{Y}^M \dot{Y}^N = \mathcal{U} , \tag{3.7}$$

where $\mathcal{U}$ is given by (2.21) and

$$\mathcal{A}_N = \frac{1}{\gamma^{00}} \left(\gamma^{0m} \Lambda_m^\mu g_{N\mu} - \frac{\Lambda_1^\mu b_{N\mu}}{\sqrt{-\gamma}} \right) \tag{3.8}$$

coincides with (2.23) for $N = a$.

Till now, all is much like before, and putting $\dot{Y}^\mu = 0$ in the equalities based on the new ansatz (1.2), we will retrieve those, which follow from the previous one ((1.1)).

Now we are going to eliminate the variables $\dot{Y}^\mu$ from (3.6) and (3.7). To this end, we express $\dot{Y}^\mu$ through $\dot{Y}^a$ from the conservation laws (3.5):

$$\dot{Y}^\mu = -\frac{\gamma^{0n}}{\gamma^{00}} \Lambda_n^\mu - \left(g^{-1}\right)^{\mu\nu} \left[g_{\nu a} \dot{Y}^a + \frac{1}{\gamma^{00} T \sqrt{-\gamma}} \left(P_\nu - T \Lambda_1^\rho b_{\nu\rho}\right) \right] . \tag{3.9}$$

After using (3.9) and (2.19), the equations of motion (3.6) and the constraint (3.7) acquire the form

$$h_{ab} \ddot{Y}^b + \Gamma^{\mathbf{h}}_{a,bc} \dot{Y}^b \dot{Y}^c = \frac{1}{2} \partial_a \mathcal{U}^{\mathbf{h}} + 2 \partial_{[a} \mathcal{A}^{\mathbf{h}}_{b]} \dot{Y}^b , \tag{3.10}$$

$$h_{ab} \dot{Y}^a \dot{Y}^b = \mathcal{U}^{\mathbf{h}} , \tag{3.11}$$

where a new, effective metric appeared

$$h_{ab} = g_{ab} - g_{a\mu} (g^{-1})^{\mu\nu} g_{\nu b} .$$

$\Gamma^{\mathbf{h}}_{a,bc}$ is the symmetric connection corresponding to this metric

$$\Gamma^{\mathbf{h}}_{a,bc} = \frac{1}{2} \left(\partial_b h_{ca} + \partial_c h_{ba} - \partial_a h_{bc} \right) .$$

The new effective scalar and gauge potentials, expressed through the background fields, are as follows

$$\mathcal{U}^{\mathbf{h}} = \frac{1}{\gamma \left(\gamma^{00}\right)^2} \left[\Lambda_1^\mu \Lambda_1^\nu g_{\mu\nu} + \frac{1}{T^2} \left(P_\mu - T \Lambda_1^\rho b_{\mu\rho}\right) (g^{-1})^{\mu\nu} \left(P_\nu - T \Lambda_1^\lambda b_{\nu\lambda}\right) \right] ,$$

$$\mathcal{A}^{\mathbf{h}}_a = -\frac{1}{\gamma^{00} T \sqrt{-\gamma}} \left[g_{a\mu} (g^{-1})^{\mu\nu} \left(P_\nu - T \Lambda_1^\rho b_{\nu\rho}\right) + T \Lambda_1^\rho b_{a\rho} \right] .$$

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We point out the qualitatively different behaviour of the potentials $\mathcal{U}^{\mathbf{h}}$ and $\mathcal{A}_a^{\mathbf{h}}$, compared to $\mathcal{U}$ and $\mathcal{A}_a$, due to the appearance of the inverse metric $(g^{-1})^{\mu\nu}$.

Since the equations (2.22), (2.20) and (3.10), (3.11) have the same form, for obtaining exact string solutions, we can proceed as before and use the previously derived formulas after the replacements $(g, \Gamma, \mathcal{U}, \mathcal{A}) \to (h, \Gamma^{\mathbf{h}}, \mathcal{U}^{\mathbf{h}}, \mathcal{A}^{\mathbf{h}})$. In particular, the solution depending on one of the coordinates X^a will be

$$\tau\left(X^a\right) = \tau_0 \pm \int_{X_0^a}^{X^a} dx \left(\frac{\mathcal{U}^{\mathbf{h}}}{h_{aa}}\right)^{-1/2} . \tag{3.12}$$

In this case by integrating (3.9), and replacing the solution for Y^μ in the ansatz (1.2), one obtains the solution for the string coordinates X^μ:

$$X^\mu(X^a,\sigma) = X_0^\mu + \Lambda_1^\mu \left[\sigma - \frac{\gamma^{01}}{\gamma^{00}}\tau\left(X^a\right)\right] - \int_{X_0^a}^{X^a} (g^{-1})^{\mu\nu} \left[g_{\nu a} \pm \frac{(P_\nu - T\Lambda_1^\rho b_{\nu\rho})}{\gamma^{00}T\sqrt{-\gamma}} \left(\frac{\mathcal{U}^{\mathrm{h}}}{h_{aa}}\right)^{-1/2}\right] dx\,. \tag{3.13}$$

To be able to take the tensionless limit $T \to 0$ in the above formulas, we have to use the λ-parameterization (2.34) of γ^{mn}. The quantities that appear in the reduced equations of motion and constraint (3.10) and (3.11), which depend on this parameterization, are $\mathcal{U}^{\mathbf{h}}$ and $\mathcal{A}_a^{\mathbf{h}}$. Now, they are given by

$$\begin{aligned}
\mathcal{U}^{h,\lambda} &= -(2\lambda^0)^2 \left[T^2\Lambda_1^\mu\Lambda_1^\nu g_{\mu\nu} + (P_\mu - T\Lambda_1^\rho b_{\mu\rho})\,(g^{-1})^{\mu\nu} \left(P_\nu - T\Lambda_1^\lambda b_{\nu\lambda}\right)\right], \\
\mathcal{A}_a^{h,\lambda} &= 2\lambda^0 \left[g_{a\mu}(g^{-1})^{\mu\lambda}(P_\lambda - T\Lambda_1^\rho b_{\lambda\rho}) + T\Lambda_1^\rho b_{a\rho}\right].
\end{aligned}$$

If one sets $\lambda^1 = 0$ and $2\lambda^0 T = 1$, the *conformal gauge* results are obtained. If one puts $T = 0$ in the above equalities, they will correspond to *tensionless* strings.

4. Some applications

In the previous two sections, we described a general approach for solving the string equations of motion and constraints in the background fields $g_{MN}(x)$ and $b_{MN}(x)$, with the help of the conserved momenta P_μ, based on the ansatzes (1.1) and (1.2). In this section, as an illustration of the previously obtained general results, we will establish the correspondence with the particular cases considered in [8] in the framework of the linear ansatz (1.1), and in [1] — in the framework of the nonlinear ansatz (1.2).

In [8], the string theory background is $AdS_5 \times S^5$, with field theory dual $\mathcal{N} = 4$ SU(N) SYM in four dimensional flat space-time. Two cases was considered: pulsating strings in AdS_5 and on S^5. The metric of the AdS_5 is taken to be

$$\begin{aligned}
ds^2_{AdS_5} &= R^2\left(-\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\Omega_3^2\right), \\
d\Omega_3^2 &= \cos^2\theta d\psi^2 + d\theta^2 + \sin^2\theta d\phi^2, \qquad R^4 = \lambda\alpha'^2\,.
\end{aligned}$$

The metric on the S^5 is given by

$$ds^2_{S^5} = R^2\left(d\theta_1^2 + \sin^2\theta_1 d\psi_1^2 + \cos^2\theta_1 d\Omega_3'^2\right).$$

For the pulsating circular string in AdS_5, the following ansatz has been used

$$t = \tau, \qquad \rho = \rho(\tau), \qquad \phi = m\sigma, \qquad \theta = \frac{\pi}{2}. \tag{4.1}$$

The relevant metric seen by the string is

$$ds^2 = R^2 \left(-\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\phi^2\right). \tag{4.2}$$

Therefore, $b_{MN} = 0$ and this metric does not depend on $x^0 = t$ and $x^2 = \phi$, i.e. $X^\mu = X^{0,2}$ and $X^a = X^1$ in our notations. Comparing the ansatzes (1.1) and (4.1), one can see that the latter is particular case of the former, corresponding to

$$\Lambda_0^0 = 1, \qquad \Lambda_1^0 = \Lambda_0^2 = 0, \qquad \Lambda_1^2 = m.$$

The induced metric (2.13) is

$$G_{00} = R^2\left(\dot{\rho}^2 - \cosh^2\rho\right), \qquad G_{01} = 0, \qquad G_{11} = m^2R^2\sinh^2\rho.$$

Taking this into account, one reproduces the Nambu-Goto action, used in [8], for the case under consideration:

$$S = -m\sqrt{\lambda}\int dt \sinh\rho\sqrt{\cosh^2\rho - \dot{\rho}^2}.$$

The conserved energy, obtained from (2.18), is

$$E = -2\pi P_0 = -2\pi TR^2\sqrt{-\gamma}\gamma^{00}\cosh^2\rho = -\sqrt{-\lambda\gamma}\gamma^{00}\cosh^2\rho.$$

The background metric (4.2) depends on only one coordinate, so our string solution is given by (2.24):

$$\tau(\rho) = \tau_0 \pm \sqrt{-\gamma^{00}}\int_{\rho_0}^{\rho}\frac{d\rho}{\sqrt{2E/\sqrt{-\lambda\gamma} + \gamma^{00}\cosh^2\rho - \gamma^{11}m^2\sinh^2\rho}}.$$

In *conformal gauge*, this solution takes the form

$$\tau(\rho) = \tau_0 \pm \int_{\rho_0}^{\rho}\frac{d\rho}{\sqrt{2E/\sqrt{\lambda} - \left(\cosh^2\rho + m^2\sinh^2\rho\right)}}.$$

In the *tensionless* limit, one obtains:

$$\tau(\rho)_{T=0} = \tau_0 \pm \int_{\rho_0}^{\rho}\frac{d\rho}{\sqrt{-\cosh^2\rho + \left(\lambda^1\right)^2 m^2\sinh^2\rho}}.$$

The second case considered in [8] is based on the ansatz:

$$t = \tau, \qquad \rho = \rho(\tau), \qquad \theta_1 = \theta_1(\tau), \qquad \psi_1 = m\sigma. \tag{4.3}$$

The relevant metric is

$$ds^2 = R^2\left(-\cosh^2\rho dt^2 + d\rho^2 + d\theta_1^2 + \sin^2\theta_1 d\psi_1^2\right) \tag{4.4}$$

and it does not depend on $x^0 = t$ and $x^3 = \psi_1$. Hence in our notations $X^\mu = X^{0,3}$, $X^a = X^{1,2}$ and

$$\Lambda_0^0 = 1, \qquad \Lambda_1^0 = \Lambda_0^3 = 0, \qquad \Lambda_1^3 = m\,.$$

The induced metric (2.13) is

$$G_{00} = R^2\left(\dot\rho^2 + \dot\theta_1^{\,2} - \cosh^2\rho\right), \qquad G_{01} = 0\,, \qquad G_{11} = m^2R^2\sin^2\theta_1\,.$$

Taking this into account, one reproduces the Nambu-Goto action, used in [8], for the case at hand:

$$S = -m\sqrt{\lambda}\int dt\sin\theta_1\sqrt{\cosh^2\rho - \dot\rho^2 - \dot\theta_1^{\,2}}\,.$$

In accordance with our general considerations in section 2, we can give three types of string solutions: when θ_1 is fixed, when ρ is fixed, and without fixing any of the coordinates θ_1 and ρ, on which the background depends.

If we fix $\theta_1 = \theta_1^0 = constant$, the solution (2.24) gives

$$\tau(\rho) = \tau_0 \pm \sqrt{-\gamma^{00}}\int_{\rho_0}^{\rho}\frac{d\rho}{\sqrt{2E/\sqrt{-\lambda\gamma} + \gamma^{00}\cosh^2\rho - \gamma^{11}m^2\sin^2\theta_1^0}}\,.$$

If we fix $\rho = \rho_0 = constant$, the solution (2.24) is

$$\tau(\theta_1) = \tau_0 \pm \sqrt{-\gamma^{00}}\int_{\theta_1^0}^{\theta_1}\frac{d\theta_1}{\sqrt{2E/\sqrt{-\lambda\gamma} + \gamma^{00}\cosh^2\rho_0 - \gamma^{11}m^2\sin^2\theta_1}}\,.$$

When none of the coordinates ρ and θ_1 is kept fixed, it turns out that the conditions (2.28) on the background are fulfilled, and therefore, the solution for the two first integrals is given by (2.30):

$$\left(R^2\dot\rho\right)^2 = -D_2(\rho) \geq 0\,, \qquad \left(R^2\dot\theta_1\right)^2 = D_2(\rho) + R^2\mathcal{U}(\rho,\theta_1) \geq 0\,. \tag{4.5}$$

The arbitrary function $D_2(\rho)$ can be fixed by the condition for separation of the variables ρ and θ_1 in the equation for θ_1. Thus if we choose

$$\frac{D_2(\rho)}{R^4} - \cosh^2\rho = -d^2 = \text{constant}\,,$$

then the equations (4.5) reduce to

$$\dot\rho^2 = d^2 - \cosh^2\rho \geq 0, \qquad \dot\theta_1^2 = \tilde{d}^2 + \frac{\gamma^{11}}{\gamma^{00}}m^2\sin^2\theta_1 \geq 0\,,$$

where

$$\tilde{d}^2 = -\frac{E}{\pi T R^2\sqrt{-\gamma}\gamma^{00}} - d^2\,.$$

These equations are solved by

$$\rho(\tau) = \rho_0 \pm \int_{\tau_0}^{\tau} d\tau\sqrt{d^2 - \cosh^2\rho}, \qquad \theta_1(\tau) = \theta_1^0 \pm \int_{\tau_0}^{\tau} d\tau\sqrt{\tilde{d}^2 + \frac{\gamma^{11}}{\gamma^{00}}m^2\sin^2\theta_1}\,.$$

One can also find the orbit $\rho = \rho(\theta_1)$, which is given by the equality

$$\int_{\rho_0}^{\rho} \frac{d\rho}{\sqrt{d^2 - \cosh^2\rho}} = \pm \int_{\theta_1^0}^{\theta_1} \frac{d\theta_1}{\sqrt{\tilde{d}^2 + \frac{\gamma^{11}}{\gamma^{00}} m^2 \sin^2\theta_1}} .$$

Let us now consider a closed string, which oscillates around the center of AdS_5 [1]:[3]

$$t = t(\tau), \qquad \rho = \rho(\tau), \qquad \phi = m\sigma, \qquad \theta = \frac{\pi}{2} .$$

The metric seen by this string is the same as in (4.2). The difference is that the above ansatz is a particular case of our general ansatz (1.2), corresponding to

$$\begin{aligned} \Lambda_0^0 &= \Lambda_1^0 = \Lambda_0^2 = 0\,, \qquad \Lambda_1^2 = m\,; \\ Y^0(\tau) &= t(\tau)\,, \qquad Y^1(\tau) = \rho(\tau)\,, \qquad Y^2(\tau) = 0\,. \end{aligned}$$

The induced metric (3.1) is

$$G_{00} = R^2\left(\dot{\rho}^2 - \dot{t}^2\cosh^2\rho\right), \qquad G_{01} = 0\,, \qquad G_{11} = m^2R^2\sinh^2\rho\,.$$

Taking this into account, one can find the Nambu-Goto action, for the case under consideration:

$$S = -m\sqrt{\lambda}\int d\tau \sinh\rho\sqrt{\dot{t}^2\cosh^2\rho - \dot{\rho}^2}\,.$$

The conserved energy, obtained from (3.5), is

$$E = -2\pi P_0 = -\frac{R^2}{\alpha'}\sqrt{-\gamma}\gamma^{00}\dot{t}\cosh^2\rho\,.$$

In *conformal gauge*, this expression reduces to the one given in [1].

The background metric (4.2) depends on only one coordinate, so our string solution is given by (3.12) and (3.13):

$$\begin{aligned} \tau(\rho) &= \tau_0 \pm \sqrt{-\gamma}\gamma^{00}\int_{\rho_0}^{\rho} d\rho\left[\frac{E^2}{\lambda\cosh^2\rho} - (m\sinh\rho)^2\right]^{-1/2}, \\ X^0(\rho) &= X_0^0 \mp \int_{\rho_0}^{\rho} \frac{d\rho}{\cosh\rho\sqrt{1 - \left(\frac{m\sqrt{\lambda}}{E}\sinh\rho\cosh\rho\right)^2}}\,. \end{aligned}$$

In the *tensionless limit* one obtains:

$$\tau(\rho)_{T=0} = \tau_0 \mp \frac{\pi R^2}{\lambda^0 E}\int_{\rho_0}^{\rho} d\rho\cosh\rho, \qquad X^0(\rho)_{T=0} = X_0^0 \mp \int_{\rho_0}^{\rho}\frac{d\rho}{\cosh\rho}\,.$$

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[3] In [1], $m = 1$.

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